PAPER

On the Performance of TCM with Channel State Information in Frequency Flat Rayleigh Mobile Channels

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SUMMARY In this paper we analize the performance of Trellis Coded Modulation (TCM) schemes with coherent detection operating in a frequency flat, mobile Rayleigh fading environment, and with different knowledge levels on both the amplitude and phase fading processes (the latter is not assumed as usual to be ideally tracked), or Channel State Information (CSI). For example, whereas ideal CSI means that both the amplitude and phase fading characteristics are perfectly known by the receiver, other situations that are treated consider perfect knowledge of the amplitude (or phase) with complete disregard of the phase (or amplitude), as well as non concern on any of them. Since these are extreme cases, intermediate situations can be also defined to get extended bounds based on Chernoff which allow the phase errors, in either form of constant phase shifts or randomly distributed phase jitter, to be included in the upper bounds attainable by transfer function methods, and are applicable to multiphase/level signaling schemes. We found that when both fading characteristics are considered, the availability of CSI enhances significatively the performance. Furthermore, for non constant envelope schemes with non ideal CSI and for constant envelope schemes with phase errors, an asymmetry property of the pairwise error probability is identified. Theoretical and simulation results are shown in support of the analysis.
key words: *trellis coded modulation, channel state information, Rayleigh channel, Chernoff bound, phase ambiguities, phase jitter*

1. Introduction

In recent years, the application of efficient coded modulation schemes [1] to channels other than the purely Gaussian have been subject of extensive research. Among these, the frequency flat (or non selective) mobile Rayleigh fading channel have deserved much attention because of its power and bandwidth practical restrictions [2]-[11].

On the other hand, in order to assist to the Viterbi decoding of the received signals, it is usual to extract information (Channel State Information (CSI)) about the amplitude and phase fading processes. This is an operation closely connected to that of carrier recovery. In the literature, it is sometimes assumed that the uniformly distributed phase fading is ideally tracked by a suitable circuit, and that only the amplitude in the form of a Rayleigh distributed random process remains affecting the received signal. Consequently, availability or unavailability of CSI is referred only to the amplitude fading. Under this criterion, it has been stated that few is the gain to be paid by ideal CSI [5]. Thus, all the bounds derived in Refs. [2]-[4] are based on this model, whereas in Ref. [10], both fading characteristics are considered though leading to so called truncated bounds, wherein only a subset of dominant terms are considered. Also in Ref. [11], the two characteristics are included in the analysis but the same is restricted to the case of no amplitude CSI, while a quasi analytical method is employed to obtain upper bounds.

The main purpose of this paper is to analize the performance of TCM schemes in a system model where both the amplitude and the phase fading processes are considered. We analize the following cases: perfect knowledge of the amplitude and phase fading, perfect knowledge only about the amplitude (or the phase) while the phase (or amplitude) is fully ignored, and also complete disregard of both. For the purpose of predicting the behavior of such schemes, we apply the Chernoff bound to the pairwise error probability. While the attainable bounds may in some circumstances be considered as weak, they are still the fast and simplest to calculate, being able to provide upper bounds by transfer function techniques given that an ideal interleaver/deinterleaver is used [2]-[4], [9]. We show then that the four above situations can be reduced to two in generalized expressions, which allow the attainment of extended bounds wherein phase errors in the form of constant phase shifts or randomly distributed phase jitter can be directly specified. The paper is organized as follows. Section 2 introduces the system model. In Sect. 3, following a brief review of the transfer function bounds, we discuss the system's performance based on the availability of CSI. The phase shifts and phase jitter phenomena are treated in Sect. 4. To illustrate the validity of the analysis, theoretical and simulation results are presented in Sect. 5, and finally our conclusions are resumed in Sect. 6.

2. System Model

As shown in Fig. 1, the system model consists, at the

Manuscript received August 4, 1993.
Manuscript revised October 12, 1993.
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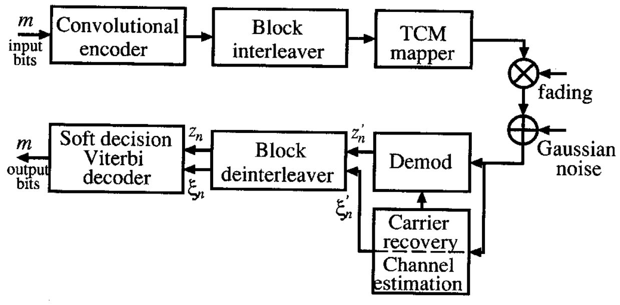

Fig. 1 System model.

transmitter side, of a convolutional encoder and a theoretically perfect block interleaver followed by a mapper which outputs a symbol on the basis of set partitioning rules. The block interleaver is used to counteract the correlation on successively faded symbols, yielding by the way a mathematically tractable model. The transmission media includes multiplicative fading as well as additive white Gaussian noise. Due to the displacement of the vehicle, the correlation evidences itself by shaping the power spectrum associated to the fading, resembling a filter with cutoff given by the maximum Doppler frequency f_m [13].

Upon arrival, the signal is input to a demodulator, and also to a circuit for carrier recovery (for accomplishing coherent detection) and for estimating the channel (to provide CSI). The outputs denoted z_n' and ξ_n' are deinterleaved to restore the original sequence order, prior to their processing by the Viterbi decoder (assumed to include a quantizer for the aim of soft-decoding). These deinterleaved counterparts are represented by z_n and ξ_n, respectively, and z_n is given as

$$z_n = c_n s_n + \eta_n \tag{1}$$

where c_n is a complex valued zero-mean stationary Gaussian random process, which characterizes the multipath fading channel. Accordingly, its Rayleigh distributed amplitude and uniformly distributed phase affect the transmitted complex valued symbol s_n, and the same is further disturbed by the zero-mean, also stationary Gaussian process $\eta_n = \eta_{Rn} + j\eta_{In}$, independent of c_n (η_{Rn} and η_{In} are independent identically distributed (i.i.d.) Gaussian processes, each with variance σ^2). The fading channel coefficient is defined as

$$c_n = \rho_n e^{j\theta_{cn}} = c_{Rn} + jc_{In} \tag{2}$$

and it follows that the non-linear transformations $c_{Rn} = \rho_n \cos\theta_{cn}$ and $c_{In} = \rho_n \sin\theta_{cn}$ yield i.i.d. Gaussian distributed processes.

Based on the availability of CSI, or on the knowledge of ρ_n and θ_{cn} by the receiver, we will treat in detail four (actually extreme) cases in the forthcoming section. Consideration of each extreme case has practical importance because they might occur if the circuits used to compensate ρ_n and/or θ_{cn} are broken down. Furthermore, they are theoretically important because the same can be used to verify the validity of the results for imperfect CSI treated in Sect. 4.

3. Error Probability Analysis

3.1 Review of Transfer Function Bounds

Let's recall the bit error probability upper bound given by

$$P_b < \frac{1}{mN} \frac{\partial \mathbf{1} \cdot \boldsymbol{T}(D, I) \cdot \mathbf{1}^T}{\partial I}\bigg|_{I=1}, \tag{3}$$

where m is the number of input bits, and N is the number of convolutional encoder states [12]. Also, $\boldsymbol{T}(D, I)$ is a $N \times N$ transfer function matrix given in terms of the parameter D (to be defined later in connection with the Chernoff bound) and of the indeterminate I (introduced to extend the capability of the transfer function in attaining bit error probability upper bounds). $\mathbf{1}$ is an N-dimensional row vector with entries all equal to 1, and $\mathbf{1}^T$ is its transpose. From Ref. [12], we can write

$$\boldsymbol{T}(D, I) = \sum_{L=1}^{\infty} \sum_{S_L} \sum_{S_L'} \boldsymbol{G}(S_L, S_L') \tag{4}$$

$$\boldsymbol{G}(S_L, S_L') \triangleq \prod_{n=1}^{L} I^{\varepsilon} \boldsymbol{G}(\boldsymbol{e}_n) \tag{5}$$

where S_L and S_L' denote the transmitted and decoded sequences, respectively, defining an error event of length L. In turn, the $N \times N$ matrix $\boldsymbol{G}(S_L, S_L')$ contains as elements, error event probability upper bounds for a given pair of sequences (S_L, S_L'). $\boldsymbol{G}(\boldsymbol{e}_n)$ is referred to as the error weight matrix, associated to the error codeword $\boldsymbol{e}_n$ assigned as a branch label to a particular transition in the N state transition diagram of the underlying convolutional encoder [12]. Finally, the exponent of I, ε, indicates the number of erroneous information bits in each transition. Computation of the upper bound in Eq. (3) by means of the matrix transfer function in Eq. (4) is particularly suited for non-uniform or error probability sequence dependent schemes, though it may be simplified to a scalar calculation when certain uniformity conditions are satisfied [12]. In any case, the p-th row, q-th column entry of the error weight matrix is obtained by

$$[\boldsymbol{G}(\boldsymbol{e}_n)]_{p,q} = \frac{1}{2^m} D = \frac{1}{2^m} E[\exp \lambda\delta_n] \tag{6}$$

where the operator $E[\cdot]$ denotes statistical expectation. $E[\exp \lambda\delta_n]$ is referred as Chernoff factor hereafter [4], and originates from the Chernoff bound (with λ as a parameter to be optimized) on the pairwise error probability $P[S_L \rightarrow S_L']$, as follows

$$P[S_L \rightarrow S_L'] = \Pr\left[\sum_{n=1}^{L} \delta_n > \mathbf{0}\right] \leq \frac{1}{2}\prod_{n=1}^{L} E[\exp \lambda\delta_n] \quad (7)$$

$$\delta_n = |\beta_n|^2 - |\beta_n'|^2 \quad (8)$$

where $|\beta_n|^2$ and $|\beta_n'|^2$ are the squared distance branch metrics of S_L and S_L', respectively, under the assumption that S_L is a correct path. The one half factor in Eq. (7) tightens the Chernoff bound according to Ref. [16], provided that the underlying scheme is uniform. Furthermore, β_n and β_n' can be represented as

$$\beta_n = c_n s_n + \eta_n - \xi_n s_n \quad (9)$$

$$\beta_n' = c_n s_n + \eta_n - \xi_n s_n' \quad (10)$$

where depending on the CSI, ξ_n may assume four limiting situations which we analyze next.

3. 2 Perfect Knowledge of Both ρ_n and θ_{cn} ($\xi_n = \rho_n e^{j\theta_{cn}}$)

In this case, both the amplitude ρ_n and the phase fading θ_{cn} processes are perfectly estimated, deserving the denomination of ideal CSI. Thus, the branch metrics β_n and β_n' become

$$\beta_n = c_n s_n + \eta_n - c_n s_n = \eta_n \quad (11)$$

$$\beta_n' = c_n s_n + \eta_n - c_n s_n' = c_n e_n + \eta_n \quad (12)$$

where $e_n = s_n - s_n' = e_{R_n} + je_{I_n}$ is the complex valued branch symbol error. With the previous definitions, δ_n is obtained as

$$\delta_n = -\rho_n^2 d_n^2 - 2\rho_n(\cos\theta_{cn} e_{R_n} - \sin\theta_{cn} e_{I_n})\eta_{R_n} - 2\rho_n(\cos\theta_{cn} e_{I_n} + \sin\theta_{cn} e_{R_n})\eta_{I_n} \quad (13)$$

where $d_n^2 = e_{R_n}^2 + e_{I_n}^2$ is the branch symbol squared Euclidean distance. A usual way of obtaining the Chernoff factor of Eq. (7) is to average δ_n in Eq. (13) over η_n conditioned on both ρ_n and θ_{cn}, and then to average successively over the two laters to remove the conditions. (In the case of ideal phase tracking, e.g. [2], [3], the only condition is on ρ_n.)

Here, we show an alternative way of looking at δ_n as a single random process by investigating its probability density function. The convenience of this method is that the Chernoff factor can be obtained directly in one step, thus avoiding the need of successively averaging over η_n, ρ_n and θ_{cn}. We define $X_n = c_n e_n = X_{R_n} + jX_{I_n}$ (in Appendix A, it is shown that X_{R_n} and X_{I_n} are mutually independent processes with variances $\sigma_{X_R}^2 = \sigma_{X_I}^2 = \sigma_X^2$), and replace Eqs. (11), (12) in Eq. (8) to get

$$\delta_n = -|X_n|^2 - 2\,\mathcal{R}\{X_n^* \eta_n\} = -X_{R_n}(X_{R_n} + 2\eta_{R_n}) - X_{I_n}(X_{I_n} + 2\eta_{I_n}) \triangleq -\delta_{R_n} - \delta_{I_n} \quad (14)$$

where X_n^* stands for the complex conjugate of X_n. We can easily show that δ_{R_n} and δ_{I_n} are statistically independent.

Let now

$$\delta_{W_n} \triangleq X_{W_n}(2\eta_{W_n} + X_{W_n}) \quad (15)$$

where W_n represents either R_n or I_n. By making $\psi_{W_n} = X_{W_n} + 2\eta_{W_n}$, we get

$$f(\psi_{W_n}) = \frac{1}{\sqrt{2\pi}\,\sigma_\psi}\exp\left(-\frac{\psi_{W_n}^2}{2\sigma_\psi^2}\right) \quad (16)$$

which is a zero-mean Gaussian random process with variance $\sigma_\psi^2 = \sigma_X^2 + 4\sigma^2$. Then, from Eq. (15), δ_{W_n} is a product of two statistically dependent Gaussian distributed random processes X_{W_n} and ψ_{W_n}.

We deal now with the derivation of the probability density function (pdf) of δ_{W_n}. In principle, we use the result [14]

$$f(\delta_{W_n}) = \int_{-\infty}^{\infty} \frac{1}{|\psi_{W_n}|} g\left(\frac{\delta_{W_n}}{\psi_{W_n}}, \psi_{W_n}\right) d\psi_{W_n} \quad (17)$$

where $g((\delta_{W_n}/\psi_{W_n}), \psi_{W_n})$ is the joint pdf of X_{W_n} and ψ_{W_n}. By joint normality we obtain

$$g(X_{W_n}, \psi_{W_n}) = \frac{\exp\left[-\frac{1}{2(1-r_{X\psi}^2)}\left(\frac{X_{W_n}^2}{\sigma_X^2} - 2r_{X\psi}\frac{X_{W_n}\psi_{W_n}}{\sigma_X\sigma_\psi} + \frac{\psi_{W_n}^2}{\sigma_\psi^2}\right)\right]}{2\pi\sigma_X\sigma_\psi\sqrt{1-r_{X\psi}^2}} \quad (18)$$

where $r_{X\psi}$ is the correlation coefficient defined as

$$r_{X\psi} \triangleq \frac{E[(X_{W_n} - m_{X_W})(\psi_{W_n} - m_{\psi_W})]}{\sigma_X\sigma_\psi} = \frac{\sigma_X}{\sigma_\psi} \quad (19)$$

where the last equality holds since both X_{W_n} and ψ_{W_n} are zero-mean ($m_{X_W}=0$, $m_{\psi_W}=0$) and $\sigma_X^2 = E[X_{W_n}^2]$. Upon simplification Eq. (17) results

$$f(\delta_{W_n}) = \int_{-\infty}^{\infty} \frac{\exp\left[-\frac{1}{2(1-r_{X\psi}^2)}\left(\frac{\delta_{W_n}^2}{\psi_{W_n}^2\sigma_X^2} - 2r_{X\psi}\frac{\delta_{W_n}}{\sigma_X\sigma_\psi} + \frac{\psi_{W_n}^2}{\sigma_\psi^2}\right)\right]}{|\psi_{W_n}| 2\pi\sigma_X\sigma_\psi\sqrt{1-r_{X\psi}^2}} \cdot d\psi_{W_n}$$

$$= \frac{\exp\left[\frac{r_{X\psi}\delta_{W_n}}{(1-r_{X\psi}^2)\sigma_X\sigma_\psi}\right]}{\pi\sigma_X\sigma_\psi\sqrt{1-r_{X\psi}^2}} \cdot \underbrace{\int_0^{\infty} \frac{\exp\left[-\frac{1}{2(1-r_{X\psi}^2)}\left(\frac{\delta_{W_n}^2}{\psi_{W_n}^2\sigma_X^2} + \frac{\psi_{W_n}^2}{\sigma_\psi^2}\right)\right]}{\psi_{W_n}} d\psi_{W_n}}_{(a)} \quad (20)$$

where the property $g(x, y) = g(-x, -y)$ is used. Let now $\tau_n=\psi_{W_n}^2$ such that $d\tau_n=2\psi_{W_n}d\psi_{W_n}$. Thus

$$(a)=\int_0^\infty \frac{1}{\psi_{W_n}}\exp\left[-\frac{1}{2(1-r_{X\varphi}^2)}\left(\frac{\delta_{W_n}^2}{\tau_n\sigma_X^2}+\frac{\tau_n}{\sigma_\varphi^2}\right)\right]\cdot\frac{1}{2\psi_{W_n}}d\tau_n$$

$$=\frac{1}{2}\int_0^\infty \frac{1}{\tau_n}e^{-p\tau_n}e^{-t^2/\tau_n}d\tau_n \quad (21)$$

where $p=\frac{1}{2(1-r_{X\varphi}^2)\sigma_\varphi^2}$ and $t^2=\frac{\delta_{W_n}^2}{2(1-r_{X\varphi}^2)\sigma_X^2}$. Now we note that

$$\int_0^\infty \frac{1}{\tau_n}e^{-p\tau_n}e^{-t^2/\tau_n}d\tau_n=\mathcal{L}[\tau_n^{-1}e^{-t^2/\tau_n}] \quad (22)$$

is just the Laplace transform ($\mathcal{L}$ denotes Laplacian operator) of $\tau_n^{-1}e^{-t^2/\tau_n}$. In Ref. [15] it can be found

$$\mathcal{L}[\tau_n^\nu e^{-t^2/\tau_n}]=2\left(\frac{t}{\sqrt{p}}\right)^{1+\nu}K_{1+\nu}(2t\sqrt{p}) \quad (23)$$

where $K_{1+\nu}$ is the $(1+\nu)$-th order nodified Bessel (or Basset, or modified Hankel, etc.) function of the third kind. In our case, $\nu=-1$ such that

$$(a)=K_0(2t\sqrt{p}) \quad (24)$$

By replacing Eq. (24) in Eq. (20) we obtain

$$f(\delta_{W_n})=\frac{\exp\left[\frac{r_{X\varphi}\sigma W_n}{(1-r_{X\varphi}^2)\sigma_X\sigma_\varphi}\right]K_0\left[\frac{|\delta_{W_n}|}{(1-r_{X\varphi}^2)\sigma_X\sigma_\varphi}\right]}{\pi\sigma_X\sigma_\varphi\sqrt{1-r_{X\varphi}^2}} \quad (25)$$

where $-\infty<\delta_{W_n}<\infty$. In Appendix B it is demonstrated that $f(\delta_{W_n})$ of Eq. (25) satisfies the probability density function definition (i.e., that $\int_{-\infty}^{\infty}f(\delta_{W_n})\,d\delta_{W_n}=1$). It follows then that the probability density function of δ_n is given by the convolution of $f(\delta_{R_n})$ and $f(\delta_{I_n})$. Unfortunately, it is not possible to obtain this pdf in closed form. Nevertheless, this will be irrelevant in the derivation of the Chernoff factor, as shown next.

Using Eq. (14), the expectation in Eq. (6) or Eq. (7) becomes

$$E[\exp\lambda\delta_n]=E[\exp\lambda'\delta_{R_n}]E[\exp\lambda'\delta_{I_n}] \quad (26)$$

where $\lambda'=-\lambda$, and the product is possible since δ_{R_n} and δ_{I_n} are statistically independent. Then, with W_n denoting either R_n or I_n,

$$E[\exp\lambda'\delta_{W_n}]=\int_{-\infty}^{\infty}e^{\lambda'\delta_{W_n}}f(\delta_{W_n})\,d\delta_{W_n} \quad (27)$$

which, with a similar manipulation to that in Appendix B, and considering further that both δ_{R_n} and δ_{I_n} are equally distributed, is obtained as

$$E[\exp\lambda\delta_n]=\{E[\exp\lambda'\delta_{W_n}]\}^2$$

$$=\frac{1-r_{X\varphi}^2}{1-[\lambda'(1-r_{X\varphi}^2)\sigma_X\sigma_\varphi+r_{X\varphi}]^2} \quad (28)$$

Optimizing Eq. (28) over λ by making $dE[\exp\lambda\delta_n]/d\lambda=0$ gives $\lambda_{opt}=1/4\sigma^2$. Then, by replacing λ_{opt} in Eq. (28) we obtain

$$E[\exp\lambda_{opt}\delta_n]=1-r_{X\varphi}^2 \quad (29)$$

which yields the bound as a function of a single parameter, the correlation coefficient $r_{X\varphi}$. Further, as shown in Appendix A

$$\sigma_X^2=\frac{d_n^2\sigma_c^2}{2}=\frac{d_n^2}{2} \quad (30)$$

where $d_n^2=e_{R_n}^2+e_{I_n}^2$ and σ_c^2 is the variance of the complex fading process which is normalized to unity. Using Eqs. (19), (30) and $\sigma_\varphi^2=\sigma_X^2+4\sigma^2$, we get

$$E[\exp\lambda_{opt}\delta_n]=\frac{1}{\frac{d_n^2}{8\sigma^2}+1} \quad (31)$$

which is applicable to any signaling scheme (constant envelope e.g., MPSK, or non constant envelope as QAM). This expression coincides with Eq. (4) in Ref. [3], which treats the particular case of MPSK (where $1/2\sigma^2=E_s/N_0$) under the assumption of perfect phase tracking with the Rayleigh envelope process as the fading source. Equation (31) has been used as a guideline in the design of codes specially appropriate for the fading rather than for the Gaussian channel [3]-[8]. As shown in Ref. [3], the performance tendency at high values of signal to noise ratio E_s/N_0 is dominated by the inverse non-zero squared branch distances product along the shortest error event in the trellis.

3.3 Perfect Knowledge of ρ_n, Phase θ_{c_n} Ignored ($\xi_n=\rho_n$)

Here, while the amplitude ρ_n is exactly estimated, no information on the phase fading process θ_{c_n} is available, and consequently no compensation of the phase fading effect on the received signal is possible. In this case, we have

$$\beta_n=c_ns_n+\eta_n-\rho_ns_n \quad (32)$$

$$\beta_n'=c_ns_n+\eta_n-\rho_ns_n' \quad (33)$$

which leads to

$$\delta_n=(|s_n|^2-|s_n'|^2)\rho_n^2-2\,\mathcal{R}\{(c_ns_n+\eta_n)^*\rho_ne_n\}$$

$$=k_n\rho_n^2+(a_n\cos\theta_{c_n}+b_n\sin\theta_{c_n})\rho_n^2+\rho_n\alpha_n \quad (34)$$

$$k_n=|s_n|^2-|s_n'|^2 \quad (35)$$

$$a_n=-2(s_{R_n}e_{R_n}+s_{I_n}e_{I_n}) \quad (36)$$

$$b_n=-2(s_{R_n}e_{I_n}-s_{I_n}e_{R_n}) \quad (37)$$

$$\alpha_n=-2e_{R_n}\eta_{R_n}-2e_{I_n}\eta_{I_n} \quad (38)$$

Note that k_n represents the energy difference between the correct and incorrect signals ($k_n=0$ for constant envelope signaling), and can be shown to satisfy the following relation

$$k_n = -d_n^2 + 2\,\mathcal{R}\{s_n^* e_n\} = -d_n^2 - a_n \tag{39}$$

In the near past, employing non constant envelope schemes (e.g. QAM) in fading channels was judged as prohibitive, because of their requirements of very precise fading compensation techniques [5]. Recently however, several developments have led to a change of this mentality, and in particular QAM schemes are seen as very promising due their high spectral efficiency [17], [18].

In the current case, it is not possible to obtain an explicit expression on the probability density function of δ_n. However, it should be noted that to simplify the subsequent expectation, we have defined the Gaussian process α_n consisting of a linear transformation of the mutually independent and Gaussian distributed processes η_{Rn}, η_{In}. The variance of α_n is obtained as

$$\sigma_{\alpha n}^2 = 4 d_n^2 \sigma^2 \tag{40}$$

Then, by successively averaging over α_n and ρ_n, we get

$$E[\exp \lambda\delta_n | \theta_{cn}] = \frac{1}{1 - \frac{\lambda}{2\sigma^2}[k_n + a_n \cos\theta_{cn} + b_n \sin\theta_{cn} + \lambda d_n^2]} \tag{41}$$

where optimization of λ does not lead to a time independent quantity. Thus, it must be numerically optimized (to minimize the upper bound of Eq. (3)). Finally, Eq. (41) must be averaged over the random phase θ_{cn}, which is uniformly distributed between $-\pi$ and π. Thus, we write

$$E[\exp \lambda\delta_n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{v_n \sin\theta_{cn} + w_n \cos\theta_{cn} + x_n} d\theta_{cn} \tag{42}$$

where $v_n = -\lambda b_n/2\sigma^2$, $w_n = -\lambda a_n/2\sigma^2$ and $x_n = 1 - (\lambda/2\sigma^2)(k_n + \lambda d_n^2)$. The above integral is available from tables for the undefined limits case. For our case, the following solution is gotten

$$E[\exp \lambda\delta_n] = \frac{1}{\sqrt{x_n^2 - v_n^2 - w_n^2}} = \frac{1}{\sqrt{\left[1 - \frac{\lambda}{2\sigma^2}(k_n + \lambda d_n^2)\right]^2 - \frac{\lambda^2 |s_n|^2 d_n^2}{\sigma^4}}} \tag{43}$$

restricted to the condition $x_n^2 > v_n^2 + w_n^2$.

3.4 Perfect Knowledge of θ_{cn}, Unknown Amplitude ρ_n ($\xi_n = e^{j\theta cn}$)

We consider now the case where the phase fading process θ_{cn} is ideally tracked, whereas the amplitude ρ_n is by no means compensated.

The branch metrics under the present condition become

$$\beta_n = c_n s_n + \eta_n - e^{j\theta cn} s_n \tag{44}$$

$$\beta_n' = c_n s_n + \eta_n - e^{j\theta cn} s_n' \tag{45}$$

and after some simplification

$$\delta_n = (|s_n|^2 - |s_n'|^2 - 2\,\mathcal{R}\{(c_n s_n + \eta_n)^* e^{j\theta cn} e_n\}$$
$$= k_n + a_n \rho_n + g(\theta_{cn})\eta_{Rn} + h(\theta_{cn})\eta_{In} \tag{46}$$

where k_n and a_n are the same as previously defined, and

$$g(\theta_{cn}) = -2(\cos\theta_{cn} e_{Rn} - \sin\theta_{cn} e_{In}) \tag{47}$$

$$h(\theta_{cn}) = -2(\cos\theta_{cn} e_{In} + \sin\theta_{cn} e_{Rn}) \tag{48}$$

Chernoff bounding the pairwise error probability produces the result

$$E[\exp \lambda\delta_n] = e^{\frac{\lambda k_n}{2\sigma^2}} e^{\frac{\lambda^2 d_n^2}{2\sigma^2}} \left[1 + \frac{\lambda}{2\sigma^2} a_n \sqrt{\pi}\, e^{\frac{\lambda^2 a_n^2}{16\sigma^4}} \cdot Q\left(-\frac{\lambda}{2\sigma^2}\frac{a_n}{\sqrt{2}}\right)\right] \tag{49}$$

where $Q(\cdot)$ denotes the tail integral of the Gaussian density, also called Q function, defined by $Q(x) = (1/\sqrt{2\pi})\int_x^{\infty} e^{-t^2/2} dt$. Once more λ demands a numerical optimization. The result of Eq. (49) also coincides with that of Ref. [3] (Eq. (19)), which treats only the particular case $k_n=0$ under the Rayleigh rather than the Gaussian fading model. Thus, the assumption of perfect phase tracking plus Rayleigh amplitude fading unknown to the receiver is tantamount to assuming that the fading is Gaussian modeled from where only the phase is perfectly known.

3.5 Unknown Both Amplitude ρ_n and Phase θ_{cn} ($\xi_n = 1$)

This is the situation that arises when no information on the fading channel characteristics is used in assisting the Viterbi decoding. For the present condition

$$\beta_n = c_n s_n + \eta_n - s_n \tag{50}$$

$$\beta_n' = c_n s_n + \eta_n - s_n' \tag{51}$$

and also

$$\delta_n = (|s_n|^2 - |s_n'|^2) - 2\,\mathcal{R}\{(c_n s_n + \eta_n)^* e_n\}$$
$$= k_n + \alpha_n \tag{52}$$

$$\alpha_n = a_n c_{Rn} + b_n c_{In} - 2e_{Rn}\eta_{Rn} - 2e_{In}\eta_{In} \quad (53)$$

where α_n is now given by the linear transformation of the mutually independent and Gaussian distributed processes c_{Rn}, c_{In}, η_{Rn}, η_{In}, and a_n, b_n are the same defined before. Then, δ_n is a k_n-mean normal process such that for constant envelope schemes, i.e. $k_n=0$, it results zero-mean. In this case, $\sum_{n=1}^{L}\delta_n$ becomes as well zero mean $\left(\sum_{n=1}^{L}k_n=0\right)$ Gaussian distributed and consequently

$$\Pr\left[\sum_{n=1}^{L}\delta_n>0\right]=\frac{1}{2} \quad (54)$$

which renders a useless system's performance irrespective of the complexity of the underlying code. Thus, any pretension of recovering good quality signals disestimating the use of CSI becomes nonsense. Or alternatively stated, much is the noise immunity that can be gained if CSI is employed.

In the non constant envelope case like QAM, the pairwise error probability will depend on the sign of $\sum_{n=1}^{L}k_n$, resulting larger or less than 0.5 if the same is positive or negative, respectively. Since $\sum_{n=1}^{L}\delta_n$ is Gaussian distributed, the exact probability value can be obtained by means of the Q function.

The Chernoff factor for $k_n \neq 0$ is obtained as follows

$$E[\exp\lambda\delta_n]=\exp\left[\frac{\lambda k_n}{2\sigma^2}\right]\exp\left[\frac{\lambda^2 d_n^2}{2\sigma^2}\left(\frac{|s_n|^2}{2\sigma^2}+1\right)\right] \quad (55)$$

and once more numerical optimization of λ is required (for $k_n=0$ it can be shown that $\lambda_{opt}=0$).

At this point, it is pertinent to point out that for the ideal CSI case of Sect. 3. 1, the Chernoff factor and accordingly the pairwise error probability upper bound for any signaling scheme depends only on the squared branch distance d_n^2 (see Eq. (31)). On the other hand, the non-ideal CSI Chernoff factors given by Eqs. (43), (49) and (55) show not only dependency on d_n^2 but also on k_n (note that when $k_n \neq 0$, $|s_n|^2$ in Eqs. (43) and (31) is not a constant). This fact reveals a sort of asymmetry of the pairwise error probability for non constant envelope schemes with non ideal CSI since, by exchanging the correct symbol s_n with the incorrect s_n', k_n becomes multiplied by -1 and this leads to a different value of the Chernoff factor. In other words, for a given pair of sequences (S_L, S_L'), the probability that S_L' is decoded provided that S_L was transmitted is not equal to the probability that S_L is erroneously favored against a hypothetically correct S_L'. Thus, in the non ideal CSI context the system results non uniform or sequence dependent [12]. As a consequence, the condition required for the one half factor in Eq. (7) to hold is no longer satisfied, and one must drop it when computing the corresponding bounds.

In the next section the four cases analized above are reduced into two, and extended Chernoff bounds are derived by means of which other sources of performance degradation such as phase shifts or phase jitter can be taken into account.

4. Phase Shift and Phase Jitter

In this section, we consider that the phase fading is imperfectly known (rather than perfectly known or totally unknown) to the receiver, while the amplitude fading may be exactly estimated or on the contrary fully ignored. This permits the inclusion of additional sources of performance degradation in the theoretical analysis, explicitly, of constant phase shifts and phase jitter, which occur in practice attempting against coherent detection. This leads us to the two cases which we treat next.

4. 1 Perfect Knowledge of ρ_n, Imperfect Phase Knowledge ($\xi_n=\rho_n e^{j\hat{\theta}_{cn}}$)

The branch metrics become

$$\beta_n = c_n s_n + \eta_n - \rho_n e^{j\hat{\theta}_{cn}} s_n \quad (56)$$

$$\beta_n' = c_n s_n + \eta_n - \rho_n e^{j\hat{\theta}_{cn}} s_n' \quad (57)$$

and it follows that

$$\delta_n = (|s_n|^2-|s_n'|^2)\rho_n^2 - 2\rho_n\mathcal{R}\{(c_n s_n+\eta_n)^* e^{j\hat{\theta}_{cn}} e_n\}$$
$$= k_n\rho_n^2 + e_n(\theta_{cn}, \hat{\theta}_{cn})\rho_n^2 + \alpha_n(\hat{\theta}_{cn})\rho_n \quad (58)$$

where k_n is the same as defined before, and

$$e_n(\theta_{cn}, \hat{\theta}_{cn}) = a_n\cos\theta_{dn} + b_n\sin\theta_{dn} \quad (59)$$

$$a_n = -2(s_{Rn}e_{Rn} + s_{In}e_{In}) \quad (60)$$

$$b_n = -2(s_{Rn}e_{In} - s_{In}e_{Rn}) \quad (61)$$

$$\theta_{dn} = \theta_{cn} - \hat{\theta}_{cn} \quad (62)$$

$$\alpha(\hat{\theta}_{cn}) = g(\hat{\theta}_{cn})\eta_{Rn} + h(\hat{\theta}_{cn})\eta_{In} \quad (63)$$

$$g(\hat{\theta}_{cn}) = -2(\cos\hat{\theta}_{cn}e_{Rn} - \sin\hat{\theta}_{cn}e_{In}) \quad (64)$$

$$h(\hat{\theta}_{cn}) = -2(\cos\hat{\theta}_{cn}e_{In} + \sin\hat{\theta}e_{Rn}) \quad (65)$$

We can further rewrite $e_n(\theta_{cn}, \hat{\theta}_{cn})$ by noting that

$$s_n^*(s_n - s_n') = (s_{Rn}e_{Rn} + s_{In}e_{In}) + j(s_{Rn}e_{In} - s_{In}e_{Rn}) \quad (66)$$

from where a_n and b_n can be identified as

$$a_n = -2\,\mathcal{R}\{s_n^*(s_n - s_n')\} \quad (67)$$

$$b_n = -2\,\mathcal{I}\{s_n^*(s_n - s_n')\} \quad (68)$$

which obviously means that a_n and b_n are in quadrature phase. Defining an angle, say ϕ_n, satisfying ϕ_n

$=\arctan a_n/b_n$, we get

$$a_n=\sqrt{(a_n^2+b_n^2)}\cos\phi_n \tag{69}$$

$$b_n=\sqrt{(a_n^2+b_n^2)}\sin\phi_n \tag{70}$$

and replace Eqs. (69) and (70) in Eq. (59) to obtain

$$\begin{aligned}e_n(\theta_{dn})&=\sqrt{(a_n^2+b_n^2)}\cos(\theta_{dn}-\phi_n)\\&=2|s_n^*(s_n-s_n')|\cos(\theta_{dn}-\phi_n) &(71)\\&=2|s_n|d_n\cos(\theta_{dn}-\phi_n) &(72)\end{aligned}$$

where Eq. (71) follows from Eqs. (67) and (68), and Eq. (72) comes from noting that $|s_n^*(s_n-s_n')|=|s_n^*||e_n|=|s_n|d_n$. Also, we have changed $e_n(\theta_{cn},\hat{\theta}_{cn})$ by $e_n(\theta_{dn})$ since, as can be seen from Eq. (59), the former is actually a function of the phase difference θ_{dn}. This phase difference may be a constant quantity (a phase shift in practice), or also a random variate (a phase jitter or noisy phase reference). We will consider this shortly.

The Chernoff factor for the present case (conditioned on θ_{dn}) can be shown to be

$$E[\exp\lambda\delta_n|\theta_{dn}]=\frac{1}{1-\dfrac{\lambda}{2\sigma^2}\left[k_n+2|s_n|d_n\cos(\theta_{dn}-\phi_n)+\lambda d_n^2\right]} \tag{73}$$

which is directly applicable for the case of constant phase shift, by replacing the given phase shift value in place of θ_{dn}.

On the other hand, when the analysis considers degradation by a noisy phase reference, the above equation must be averaged over a randomly distributed variable θ_{dn}. As in Refs. [19], [20], we assume that θ_{dn} is zero-mean, Gaussian distributed, and with variance $\sigma_{\theta dn}^2$ such that

$$p(\theta_{dn})=\frac{1}{\sqrt{2\pi}\,\sigma_{\theta dn}}\exp\left[-\frac{\theta_{dn}^2}{2\sigma_{\theta dn}^2}\right]. \tag{74}$$

For the particular case where $\theta_{dn}=0$, or $\hat{\theta}_{cn}=\theta_{cn}$, it can be seen from Eq. (59) that $e_n(\theta_{dn})$ reduces to a_n. By replacing Eq. (39) in Eq. (73) and averaging over θ_{dn} (whose probability density function is now a unit area impulse defined by $p(\theta_{dn})=\delta(\theta_{dn})$), we arrive (with $\lambda_{opt}=1/4\sigma^2$) to the same expression given by Eq. (31). Similarly, for $\theta_{dn}=\theta_{cn}$ or $\hat{\theta}_{cn}=0$, we arrive to the case described in Sect. 3. 2. Of course, in any of these two limiting cases, it is preferable to use Eq. (31) or Eq. (43).

4. 2 Complete Unknowledge of ρ_n, Imperfect Phase Knowledge ($\xi_n=e^{j\theta_{cn}}$)

Here we have

$$\beta_n=c_ns_n+\eta_n-e^{j\hat{\theta}_{cn}}s_n \tag{75}$$

$$\beta_n'=c_ns_n+\eta_n-e^{j\hat{\theta}_{cn}}s_n' \tag{76}$$

and δ_n is given by

$$\begin{aligned}\delta_n&=(|s_n|^2-|s_n'|^2)-2\,\mathcal{R}\{(c_ns_n+\eta_n)^*e^{j\hat{\theta}_{cn}}e_n\}\\&=k_n+e_n(\theta_{dn})\rho_n+\alpha_n(\hat{\theta}_{cn}) &(77)\end{aligned}$$

with k_n and $\alpha_n(\hat{\theta}_{cn})$ as already defined. By successively averaging over $\alpha_n(\hat{\theta}_{cn})$ (a Gaussian process) and over ρ_n, the Chernoff factor becomes

$$\begin{aligned}E[\exp\lambda\delta_n|\theta_{dn}]&=e^{\frac{\lambda k_n}{2\sigma^2}}\,e^{\frac{\lambda^2d_n^2}{2\sigma^2}}\Bigg[1+\frac{\lambda}{\sigma^2}|s_n|d_n\cos(\theta_{dn}-\phi_n)\\&\cdot\sqrt{\pi}\exp\left[\frac{\lambda^2|s_n|^2d_n^2\cos^2(\theta_{dn}-\phi_n)}{4\sigma^4}\right]\\&\cdot Q\left(-\frac{\lambda}{\sigma^2}\frac{|s_n|d_n\cos(\theta_{dn}-\phi_n)}{\sqrt{2}}\right)\Bigg] &(78)\end{aligned}$$

and as before, Eq. (78) can be used directly to calculate performance bounds with the added degradation coming from phase shift, or it can be averaged over θ_{dn} for the case of phase jitter sensitivity analysis. Since $\theta_{dn}=0$ (or $\hat{\theta}_{cn}=\theta_{cn}$) and $\theta_{dn}=\theta_{cn}$ (or $\hat{\theta}_{cn}=0$) are the two particular cases considered in Sects. 3. 3 and 3. 4, respectively, Eq. (78) is a general expression for them.

On the other hand, inspection of Eqs. (73) and (78) shows that the asymmetry property appears in these cases also for constant envelope schemes since even though $k_n=0$, the angle $\phi_n=\arctan(a_n/b_n)$ is not kept constant by exchanging s_n by s_n'. Thus, the schemes become non uniform preventing the use of the one half factor in Eq. (7).

5. Results

In this section we present some numerical results. In Fig. 2, the error event probability for one path of the 8

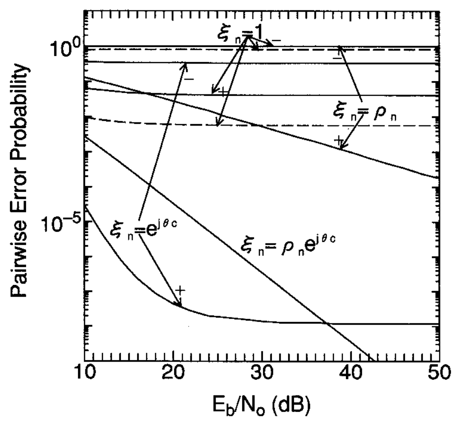

Fig. 2 One path pairwise error probability (8 state 16 QAM).

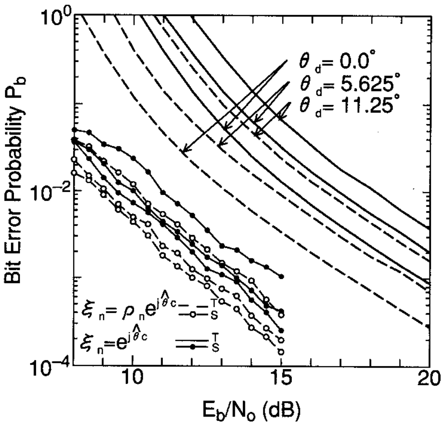

Fig. 3 Phase shift degradation (8 state 8 PSK) (T: theoretical, S: simulation).

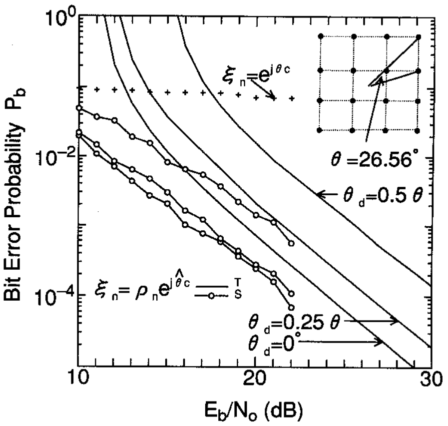

Fig. 4 Phase shift degradation (8 state 16 QAM) (T: theoretical, S: simulation).

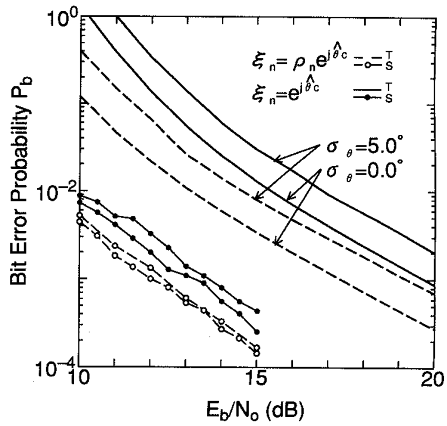

Fig. 5 Phase jitter degradation (8 state 8 PSK) (T: theoretical S: simulation).

state 16 QAM Du-Vucetic code is given, in order to illustrate the asymmetry property of the error event probability for non constant envelope schemes, as analyzed in Sect. 3. This feedback code has been specially designed for the fading channel under the assumption of ideal CSI, and is specified by the octal coefficients $h_3=16$, $h_2=04$, $h_1=07$, $h_0=13$ [7]. The arbitrarily selected error event is defined by an assumed correct sequence $(s_{R_1}=1,\ s_{I_1}=1)$, $(s_{R_2}=1,\ s_{I_2}=1)$ and by an incorrect one given by $(s'_{R_1}=3,\ s'_{I_1}=3)$, $(s'_{R_2}=-3,\ s'_{I_2}=-3)$. The notation (+) and (−) stands for the originaly correct sequence and for its permutation, respectively. We must stress here that while for the ideal CSI case, or $\xi_n=\rho_n e^{j\theta_{cn}}$, the optimum value of $\lambda=1/4\sigma^2$ is the same for one path and for the ensemble of paths which conform the union bound, this is not valid for the non ideal cases shown. That is, the optimum λ obtained numerically to minimize the Chernoff bounded error event probability for the selected path is not the same than the one which would be obtained to minimize the union bound. This explains why it seems that non ideal CSI is doing better than perfect CSI. The asymmetry property for non ideal CSI is apparent, since permutation of the correct and incorrect sequences give different error event probability results. For $\xi_n=1$ and $\xi_n=\rho_n$ the loosest Chernoff bound for the (−) sequence is gotten (corresponding to $\lambda_{opt}=0$). Since in the former case the exact pairwise probability can be computed also (as pointed out in Sect. 3. 5) by means of the Q function, we show those results with dotted lines.

The figures that follow next show both theoretical and simulation results, appearing in corresponding order. In Fig. 3, the 2/3 rate 8 state 8 PSK Ungerboeck code is evaluated for several values of phase shift, and for two conditions of CSI. As expected, the performance of the system worsens when the amplitude fading process is unknown to the receiver. However, since such process can only dilate or contract a signal vector without taking it out from its decision region, the associated degradation does not reach higher levels. Furthermore, the performance of the same above Du-Vucetic code under given phase shift values is shown in Fig. 4. In this case, the performance is negatively affected by a constant phase shift. The theoretical upper bounds for the condition $\xi_n=e^{j\theta_{cn}}$ are not shown since the degradation suffered in this case is too high for them to be considered useful. To corroborate this we show the result of the simulation when the phase error θ_{dn} is zero or equivalently $\xi_n=e^{j\theta_{cn}}$. It should be noted that a multilevel scheme as QAM is quite sensitive to the amplitude fading, since in this case the signal vector can easily reach a neighbor's decision region. Finally, assessment is made of the Ungerboeck code when failure in tracking the phase fading is noisy, and for two different CSI conditions. The performance of that scheme shown in Fig. 5, is seen degraded for the given phase standard deviation.

Although in all cases our theoretical results follow the tendency of the simulations, there is a difference between them which is typically observed when the Chernoff bound is applied to fading channels. Reasons behind are the non maximum likelihood metric used for non ideal CSI [9] and also the parameter λ which in such cases is optimized numerically for the whole ensemble of error events. We have also seen that the one half factor cannot always be used to tighten the bounds. However, in combination with transfer function techniques the Chernoff bound is a fast and simple way of providing error probability upper bounds.

6. Conclusions

We have analyzed the performance of TCM systems in frequency flat mobile Rayleigh fading channels, by considering the availability of Channel State Information (CSI). First, we analyzed four cases which can be seen as limiting situations. We have found that CSI enhances significatively the system's performance when both fading characteristics are considered. Also, an asymmetry property of the pairwise error probability has been identified for non constant envelope schemes with non ideal CSI and for constant envelope schemes with phase errors. Furthermore, extended expressions of the Chernoff factors were obtained where it is possible to directly specify phase shift values or Gaussian distributed phase jitter variances, thus yielding more useful Chernoff bound based upper bounds on the bit error probability.

Acknowledgment

The first of the authors would like to thank Prof. Ikuo Oka for his encouragements. Also to the graduate students study group of the Information Theory Laboratory for useful discussions.

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Appendix A

From the definition of X_n we have

$$X_{Rn} = c_{Rn} e_{Rn} - c_{In} e_{In} \tag{A·1}$$

$$X_{In} = c_{In} e_{Rn} + c_{Rn} e_{In} \tag{A·2}$$

Then, the covariance is given as follows

$$E[(X_{Rn} - \bar{X}_{Rn})(X_{In} - \bar{X}_{In})]$$

$$= E[X_{Rn} X_{In}] \tag{A·3}$$

$$= e_{Rn}^2 E[c_{Rn} c_{In}] - e_{Rn} e_{In} E[c_{In}^2] + e_{Rn} e_{In} E[c_{Rn}^2] - e_{In}^2 E[c_{In} c_{Rn}] \tag{A·4}$$

$$=e_{Rn}^2\cdot 0-e_{Rn}e_{In}\sigma_{c_I}^2+e_{Rn}e_{In}\sigma_{c_R}^2-e_{In}^2\cdot 0=0 \quad (\text{A}\cdot 5)$$

where Eq. (A·3) follows from considering that c_{Rn} and c_{In} are zero-mean. Equation (A·4) uses the definitions of X_{Rn} and X_{In}, while Eq. (A·5) is obtained from the uncorrelation of the real and imaginary parts of c_n, and from the fact that their respective variances are equal ($\sigma_{c_R}^2=\sigma_{c_I}^2=\sigma_c^2/2$) [13]. Thus, X_{Rn} and X_{In} are not correlated. Now, we obtain the variances of X_{Rn}, X_{In} as

$$\sigma_{X_R}^2=e_{Rn}^2\sigma_{c_R}^2+e_{In}^2\sigma_{c_I}^2 \quad (\text{A}\cdot 6)$$

$$\sigma_{X_I}^2=e_{Rn}^2\sigma_{c_I}^2+e_{In}^2\sigma_{c_R}^2 \quad (\text{A}\cdot 7)$$

$$\sigma_{X_R}^2=\sigma_{X_I}^2=\sigma_X^2=\frac{d_n^2\sigma_c^2}{2}. \quad (\text{A}\cdot 8)$$

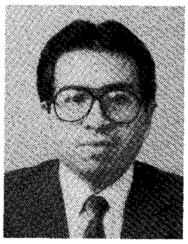
Appendix B

By splitting Eq. (25) in two terms and integrating over δ_{W_n}

$$\int_{-\infty}^{\infty} f(\delta_{W_n})\, d\delta_{W_n}=\frac{\sigma_X\sigma_\varphi(1-r_{X\varphi}^2)}{\pi\sigma_X\sigma_\varphi\sqrt{1-r_{X\varphi}^2}}\cdot\left(\int_0^\infty e^{-r_{X\varphi}w^-}K_0[w^-]dw^- +\int_0^\infty e^{r_{X\varphi}w^+}K_0[w^+]dw^+\right) \quad (\text{A}\cdot 9)$$

where $w^-=-\delta_{W_n}/(1-r_{X\varphi}^2)$ and $w^+=\delta_{W_n}/(1-r_{X\varphi}^2)$. The integrals in Eq. (A·9) can be solved also by Laplace transformation [15] and by using the identity $\arccos(x)+\arccos(-x)=\pi$

$$\int_{-\infty}^{\infty} f(\delta_{W_n})\, d\delta_{W_n}=\frac{\sigma_X\sigma_\varphi(1-r_{X\varphi}^2)}{\pi\sigma_X\sigma_\varphi\sqrt{1-r_{X\varphi}^2}}\cdot\frac{1}{\sqrt{1-r_{X\varphi}^2}}\cdot(\arccos(r_{X\varphi})+\arccos(-r_{X\varphi}))=1. \quad (\text{A}\cdot 10)$$



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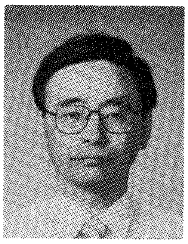

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