PAPER

The Performance of the New Convolutional Coded ARQ Scheme for Moderately Time-Varying Channels

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SUMMARY The performance of the hybrid-ARQ scheme with a convolutional code, in which the retransmission criterion is based on an estimated decoding error rate, is evaluated for moderately time-varying channels. It is shown by computer simulations that the simple average diversity combining scheme can almost attain the same performance as the optimally weighted diversity combining scheme. For the whole and partial retransmission schemes with the average diversity combining, the theoretical bounds of throughput and bit error rate are derived, and it is shown that their bounds are tight and the treated schemes can attain a given error rate with good throughput for moderately time-varying channels. Furthermore, the throughput is shown to be improved by the partial retransmission scheme compared with the whole retransmission scheme.

key words: *time-varying channel, ARQ, convolutional code, Viterbi decoding, AD/WD packet combining*

1. Introduction

In hybrid Automatic Repeat Request (ARQ) schemes with convolutional codes [1]–[7], retransmission criteria are not based on decoding error rate directly. Hence it is difficult to satisfy a given error rate for a time-varying channel by such schemes. In contrast with such schemes, we showed in the previous work [8] that the following hybrid-ARQ scheme can attain a given error rate and good throughput for any signal to noise ratio (SNR) with a fixed convolutional code because the retransmission criterion is based on an estimated decoding error rate and the packet combining technique is used.

Let $X=(x_1, x_2, \cdots, x_N)$ be a convolutionally coded packet with length N, and let each x_j be transmitted by the BPSK with the transmission power $\mathcal{E}$, i.e. $x_j=\pm\sqrt{\mathcal{E}}$. When the same packet X is transmitted I-times, the i-th received packet is represented by $Y^i=(y_1^i, y_2^i, \cdots, y_N^i)$, $i=1,2,\cdots,I$. Note that $I=1$ means no retransmission. All these received packets Y^i are combined by

$$\tilde{y}_j^I = (y_j^1 + y_j^2 + \cdots + y_j^I)/I \tag{1}$$

to make a combined packet $\tilde{Y}_{AD}^I = (\tilde{y}_1^I, \tilde{y}_2^I, \cdots, \tilde{y}_N^I)$. Since $y_j^i = x_j + n_j^i$ holds for the Gaussian noise n_j^i, each element of the combined packet $\tilde{Y}_{AD}^I$ is represented by

$$\begin{aligned}\tilde{y}_j^I &= x_j + \frac{n_j^1 + n_j^2 + \cdots + n_j^I}{I} \\ &\triangleq x_j + \tilde{n}_{ADj}^I\end{aligned} \tag{2}$$

with $\tilde{n}_{ADj}^I \triangleq (n_j^1 + n_j^2 + \cdots + n_j^I)/I$. We assume that the SNR of the i-th transmission is η_i, i.e. n_j^i is the Gaussian random variable with mean 0 and variance $\frac{\mathcal{E}}{2\eta_i}$. Hence $\tilde{n}_{ADj}^I$ becomes the Gaussian random variable with mean 0 and variance $\frac{\mathcal{E}}{2I^2}\sum_{i=1}^{I}\frac{1}{\eta_i}$. This combined packet $\tilde{Y}_{AD}^I$ is decoded by the Viterbi decoder to obtain an estimated transmitted packet $\overline{X}^I = (\overline{x}_1^I, \overline{x}_2^I, \cdots, \overline{x}_N^I)$. From $\tilde{Y}_{AD}^I$ and $\overline{X}^I$, the SNR $\overline{\eta}_I$ of the equivalent channel such that $\tilde{Y}_{AD}^I$ is received when $\overline{X}^I$ is transmitted can be estimated by

$$\overline{\eta}_I = \frac{\mathcal{E}^2}{2\sum_{j=1}^{N}\frac{(\overline{x}_j^I\tilde{y}_j^I - \mathcal{E})^2}{N}}. \tag{3}$$

If this $\overline{\eta}_I$ is greater than a given threshold T, then $\overline{X}^I$ is accepted as a decoded packet. Otherwise, the retransmission of X is requested. Note that since the error rate is determined by the SNR of the channel, "$\overline{\eta}_I \geqq T$" means that the error rate determined from T can be approximately attained. In Ref. [8], the performance of the above scheme was analyzed for time-invariant or slowly time-varying channels, i.e. for the case that η_i can be assumed to be constant during a sequence of retransmission. In this paper, we extend the above scheme to moderately time-varying channels and we show that it can also attain good performance for the Gaussian moderately time-varying channels. "Moderately time-varying" means that the SNR η of the channel can be considered to be constant in one packet Y^i, but it varies every packet transmission according to a probability distribution $p(\eta)$.

The combining scheme given by Eq. (1) is called the average diversity combining (ADC) scheme [5], [9], which provides the maximum-likelihood decoding for the time-invariant or slowly time-varying channels. However, the ADC scheme is no longer optimal for the moderately time-varying channels. Hence we also consider the following weight diversity combining (WDC)

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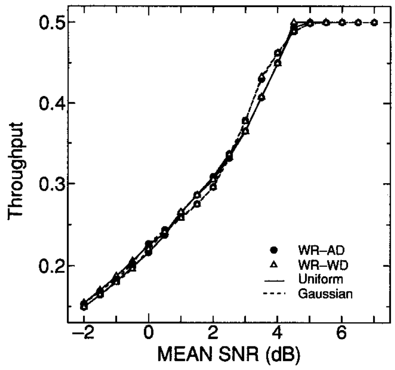

Fig. 1 Simulation result of the throughput (uniform distribution with width 4 dB and Gaussian distribution with variance 1 dB).

scheme [5], [9]. In the WDC scheme, a received packet Y^i is decoded by the Viterbi decoder to estimate the SNR $\overline{\eta}_i$ of the channel by Eq. (3). Then a combined packet $\tilde{Y}_{WD}^I = (\tilde{y}_1^I, \tilde{y}_2^I, \cdots, \tilde{y}_N^I)$ is obtained by weighting Y^i with the estimated channel noise variance $\overline{\sigma}_i^2 = \frac{\mathcal{E}}{2\overline{\eta}_i}$ as follows.

$$\begin{aligned}\tilde{y}_j^I &= \left(\frac{y_j^1}{\overline{\sigma}_1^2} + \frac{y_j^2}{\overline{\sigma}_2^2} + \cdots + \frac{y_j^I}{\overline{\sigma}_I^2}\right) \frac{1}{\sum_{i=1}^{I} \frac{1}{\overline{\sigma}_i^2}} \\ &= x_j + \left(\frac{n_j^1}{\overline{\sigma}_1^2} + \frac{n_j^2}{\overline{\sigma}_2^2} + \cdots + \frac{n_j^I}{\overline{\sigma}_I^2}\right) \frac{1}{\sum_{i=1}^{I} \frac{1}{\overline{\sigma}_i^2}} \\ &\stackrel{\triangle}{=} x_j + \tilde{n}_{WDj}^I \end{aligned} \tag{4}$$

where $\tilde{n}_{WDj}^I$ denotes the Gaussian random variable with mean 0 and variance $\left(\frac{\mathcal{E}}{2}\sum_{i=1}^{I}\frac{1}{\overline{\sigma}_i^4 \eta_i}\right) \cdot \left(\sum_{i=1}^{I}\frac{1}{\overline{\sigma}_i^2}\right)^{-2}$. By applying the Viterbi decoder to $\tilde{Y}_{WD}^I$, we obtain the estimated transmitted packet $\overline{X}^I$ and the corresponding estimated SNR $\overline{\eta}_I$. Finally it is determined, based on $\overline{\eta}_I$, whether the $\overline{X}^I$ is accepted as the decoded packet or the retransmission of X is requested.

From Eqs. (2) and (4), it is clear that $\tilde{y}_{ADj}^I$ is nearly equal to $\tilde{y}_{WDj}^I$ when the variance of probability distribution $p(\eta)$ is not large, and hence we can expect that the ADC scheme can achieve the almost same performance as the WDC scheme for such case. This expectation is confirmed by the computer simulations denoted by WR-ADC and WR-WDC in Fig. 1, in which the probability distributions $p(\eta)$ of the channel SNR are the uniform distribution with width 4 dB and the Gaussian distribution with variance 1 dB, respectively, and throughputs are plotted as the function of the mean of $p(\eta)$. Furthermore, even for huge variance case shown in Table 1, the degradation of the throughput of the ADC scheme compared with the WDC scheme is small†. On the other hand, the WDC scheme requires one additional Viterbi decoding for each retransmitted packet to estimate the channel noise variance, and hence the load of the Viterbi decoder is twice as much as the ADC scheme when the retransmission is occurred. Therefore, the ADC scheme is practically preferable than the WDC scheme. From this reason, we evaluate the performance only for the ADC scheme in this paper to save the space although the similar analysis is possible for the WDC scheme, too.

Table 1 Throughput for large variance of $p(\eta)$ (uniform distribution with width 10 dB and Gaussian distribution with variance 4 dB).

$p(\eta)$ / mean of SNR	Gaussian		Uniform	
	ADC	WDC	ADC	WDC
−2 dB	0.144	0.158	0.136	0.170
1 dB	0.260	0.266	0.251	0.267
4 dB	0.409	0.409	0.368	0.368

If the estimated $\overline{\eta}_I$ is slightly less than a given threshold T, we can attain the threshold by retransmitting some parts of a packet instead of the whole packet. Such partial retransmission (PR) scheme achieves better throughput than the whole retransmission (WR) scheme [8]. Assume that a packet of length N is divided into U subblocks, each of which contains K bits, i.e. $N = UK$. The retransmission region of $\overline{\eta}_I$, i.e. $\overline{\eta}_I < T$, is also divided into K regions, which are defined as $R_l \stackrel{\triangle}{=} \{\overline{\eta} : T_{l-1} \leqq \overline{\eta}_I < T_l\}$, $l = 1, 2, \cdots, K$, where $T_0 = T$ and $T_K = -\infty$. If the estimated SNR $\overline{\eta}_I$ is in retransmission region R_l, then l bits in each subblock are retransmitted. In each subblock, these l bits are selected in such a way that the selected bits have as same interval as possible, and they are shifted every retransmission. In this PR scheme, all received packets are combined as

$$\tilde{y}_j^I = \left(\sum_{i=1}^{I} \delta_j^i y_j^i\right) / \sum_{i=1}^{I} \delta_j^i, \tag{5}$$

where δ_j^i is defined as

$$\delta_j^i = \begin{cases} 1, & \text{if the } i\text{-th packet contains the } j\text{-th bit of } X, \\ 0, & \text{otherwise.} \end{cases}$$

See Ref. [8] for more details of the PR scheme.

Since discontinuous bits of the whole packet are retransmitted in the PR scheme, the variance of the channel noise cannot be estimated from the decoding process, which means that the WDC scheme cannot be applied to the PR scheme. The performance of the PR-ADC scheme is considered in Sect. 3.

In Sect. 4, we confirm the validity of the theoretical bounds of error rates and throughputs, which are

† The reason why we compare the throughput rather than the error rate is described in the appendix.

derived in Sects. 2 and 3, by comparing them with computer simulations, and it is shown that the considered schemes can attain good performance for the moderately time-varying channels, too.

2. Performance Evaluation of WR-ADC Scheme

2.1 Average Number of Transmission

The performance can be evaluated theoretically in the same way as shown in Ref. [8] except that it must be averaged with time-varying SNR η, i.e. $p(\eta)$. Let M_{AD} be the average number of transmission in the WR-ADC scheme and let F_I, S_I, and η_i be defined as

F_I Event that retransmission is requested for $\tilde{Y}^I$,

S_I Event that decoded packet of $\tilde{Y}^I$ is accepted,

η_i Channel SNR for the i-th transmission.

Then, M_{AD} can be bounded from Eqs. (5) and (6) in Ref. [8] as follows.

$$\begin{aligned} M_{AD} \geqq 1 &+ \int_{-\infty}^{\infty} p(\eta_1)P(F_1)d\eta_1 \\ &+ \iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(F_1)P(F_2)d\eta_1 d\eta_2 \\ &+ \iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3) \\ &\cdot P(F_1)P(F_2)P(F_3)d\eta_1 d\eta_2 d\eta_3 + \cdots. \end{aligned} \quad (6)$$

$$\begin{aligned} M_{AD} \leqq 1 &+ \int_{-\infty}^{\infty} p(\eta_1)P(F_1)d\eta_1 \\ &+ \iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(F_2)d\eta_1 d\eta_2 \\ &+ \iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P(F_3)d\eta_1 d\eta_2 d\eta_3 + \cdots. \end{aligned} \quad (7)$$

The throughput, τ_{AD}, is obtained by $\tau_{AD} = R/M_{AD}$ where R is the rate of the convolutional code.

2.2 Retransmission Probability

When the estimated SNR $\overline{\eta}_I$ of $\tilde{Y}^I$ is less than a given threshold T, retransmission is requested. Hence, the retransmission probability $P(F_I)$ can be represented by

$$P(F_I) = \sum_{\overline{X}^I} P(\overline{X}^I)P(\overline{\eta}_I < T|\overline{X}^I). \quad (8)$$

First we derive the upper bound of $P(F_I)$. By approximating the χ^2-distribution with the Gaussian distribution, we can get the upper bound of $P(F_I)$ as follows.

$$\begin{aligned} P(F_I) &< P(E_I) + \{1-P(E_I)\} P(\overline{\eta}_{I_C} < T) \\ &\lesssim P(E_I)\left[1-Q\left\{\sqrt{\frac{N}{2}}\left(\frac{I^2}{T}\frac{1}{\sum_{i=1}^{I}\frac{1}{\eta_i}}-1\right)\right\}\right] \\ &\quad +Q\left\{\sqrt{\frac{N}{2}}\left(\frac{I^2}{T}\frac{1}{\sum_{i=1}^{I}\frac{1}{\eta_i}}-1\right)\right\}, \end{aligned} \quad (9)$$

where $Q(a) \triangleq \int_a^\infty \frac{1}{\sqrt{2}} \exp\left\{-\frac{t^2}{2}\right\} dt$, E_I denotes the event that errors occur when $\tilde{Y}^I$ is decoded by the ordinary Viterbi decoder and $\overline{\eta}_{I_C}$ shows the $\overline{\eta}_I$ in the case of $\overline{X}^I = X$.

Next we derive the lower bound of $P(F_I)$. Since the events F_I and $S_I E_I$ are mutually exclusive, the following relation holds.

$$\begin{aligned} &P(F_I) + P(S_I E_I) \\ &= P(\overline{\eta}_{I_C} < T)P(F_I \cup S_I E_I|\overline{\eta}_{I_C} < T) \\ &\quad +P(\overline{\eta}_{I_C} \geqq T)P(F_I \cup S_I E_I|\overline{\eta}_{I_C} \geqq T) \\ &= P(\overline{\eta}_{I_C} < T) + \sum_{\overline{X}^I \neq X} P(\overline{\eta}_{I_C} \geqq T, \overline{X}^I) \\ &\quad \cdot P(F_I \cup S_I E_I|\overline{\eta}_{I_C} \geqq T, \overline{X}^I) \\ &\quad +P(\overline{\eta}_{I_C} \geqq T, \overline{X}^I = X) \\ &\quad \cdot P(F_I \cup S_I E_I|\overline{\eta}_{I_C} \geqq T, \overline{X}^I = X) \\ &= P(\overline{\eta}_{I_C} < T) + P(\overline{\eta}_{I_C} \geqq T, E_I), \end{aligned} \quad (10)$$

where the second equality follows from the fact that $P(F_I \cup S_I E_I|\overline{\eta}_{I_C} < T) = 1$ holds because "$\overline{\eta}_{I_C} < T$" means that either retransmission or an error occurs. Furthermore, $P(F_I \cup S_I E_I|\overline{\eta}_{I_C} \geqq T, \overline{X}^I \neq X) = 1$ and $P(F_I \cup S_I E_I|\overline{\eta}_{I_C} \geqq T, \overline{X}^I = X) = 0$ hold because "$\overline{\eta}_{I_C} \geqq T, \overline{X}^I \neq X$" means that an error occurs and "$\overline{\eta}_{I_C} \geqq T, \overline{X}^I = X$" means that neither retransmission nor error occurs. Hence the third equality of Eq. (10) holds. From Eq. (10) we obtain

$$\begin{aligned} &P(F_I) \\ &= P(\overline{\eta}_{I_C} < T) + P(\overline{\eta}_{I_C} \geqq T, E_I) - P(S_I E_I) \quad (11) \\ &= P(\overline{\eta}_{I_C} < T) + P(\overline{\eta}_{I_C} \geqq T, E_I) \\ &\quad -P(E_I) + P(F_I E_I) \\ &> P(\overline{\eta}_{I_C} < T) - P(E_I) + P(F_I)P(E_I). \quad (12) \end{aligned}$$

Hence, $P(F_I)$ can be bounded below as follows.

$$P(F_I) > \frac{P(\overline{\eta}_{I_C} < T) - P(E_I)}{1 - P(E_I)}. \quad (13)$$

However, it is difficult to derive a tight upper bound of $P(E_I)$ for low SNR. But in such low SNR, $P(S_I) \ll P(\overline{\eta}_{I_C} < T)$ and $P(\overline{\eta}_{I_C} \geqq T) \ll P(\overline{\eta}_{I_C} < T)$ usually holds. Hence Eq. (13) can be approximated very closely by $P(\overline{\eta}_{I_C} < T)$. Furthermore, when the SNR is not low, $P(\overline{\eta}_{I_C} < T) \ll P(E_I)$ holds. Hence, $P(F_I)$ can be approximated very closely by $P(\overline{\eta}_{I_C} < T)$ for any SNR.

This can be confirmed by simulations. Therefore we use the following approximation.

$$P(F_I) \approx P(\overline{\eta}_{I_C} < T). \tag{14}$$

2.3 Average Bit Error Rate of WR-ADC Scheme

The block error probability of $\tilde{Y}^I$, say P_E, can be bounded from Eq. (7) in Ref. [8] as follows.

$$\begin{aligned} P_E \leqq & \int_{-\infty}^{\infty} p(\eta_1)P(E_1)\{1-P(F_1)\}\,d\eta_1 \\ & + \iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(E_2) \\ & \cdot\{1-P(F_1)P(F_2)\}\,d\eta_1 d\eta_2 \\ & + \iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P(E_3) \\ & \cdot\{1-P(F_1)P(F_2)P(F_3)\}\,d\eta_1 d\eta_2 d\eta_3 + \cdots. \end{aligned} \tag{15}$$

Similarly, letting P_B^I be the bit error rate of $\tilde{Y}^I$ for the ordinary Viterbi decoder, the bound of the bit error rate P_B is obtained by replacing $P(E_I)$ in Eq. (15) with P_B^I as follows.

$$\begin{aligned} P_B \leqq & \int_{-\infty}^{\infty} p(\eta_1)P_B^1\{1-P(F_1)\}\,d\eta_1 \\ & + \iint_{-\infty}^{\infty} p(\eta_1\eta_2)P_B^2 \\ & \cdot\{1-P(F_1)P(F_2)\}\,d\eta_1 d\eta_2 \\ & + \iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P_B^3 \\ & \cdot\{1-P(F_1)P(F_2)P(F_3)\}\,d\eta_1 d\eta_2 d\eta_3 + \cdots. \end{aligned} \tag{16}$$

To derive P_B^I, we use the following notation.

- $\hat{X}_L$ Incorrect path which diverges from the correct path X and remerges exactly after L branches.
- $A(X, \hat{X}_L)$ The number of erroneous information bits corresponding to an incorrect path $\hat{X}_L$.
- $d = d(X, \hat{X}_L)$ Hamming distance between paths X and $\hat{X}_L$.
- $m(\tilde{Y}^I, X) = \sum_{j=1}^{N} \tilde{y}_j^I x_j$ Path metric which is used in the Viterbi decoder.

Since the mean and variance of $m(\tilde{Y}^I, X) - m(\tilde{Y}^I, \hat{X})$ are $-2\mathcal{E}d$ and $\sigma^2 = 4\mathcal{E}d\frac{1}{2I^2}\sum_{i=1}^{I}\frac{\mathcal{E}}{\eta_i}$, respectively, P_B^I is bounded as follows.

$$\begin{aligned} P_B^I < & \sum_{L=1}^{\infty}\sum_{\hat{X}_L} A(X, \hat{X}_L) \\ & \cdot P(m(\tilde{Y}^I, X) - m(\tilde{Y}^I, \hat{X}_L) < 0) \\ = & \sum_{L=1}^{\infty}\sum_{\hat{X}_L} A(X, \hat{X}_L) \\ & \cdot \int_{-\infty}^{0} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(t-2\mathcal{E}d)^2}{2\sigma^2}\right\} dt \\ = & \sum_{L=1}^{\infty}\sum_{\hat{X}_L} A(X, \hat{X}_L) Q\left(\sqrt{\frac{2I^2 d}{\sum_{i=1}^{I}\frac{1}{\eta_i}}}\right). \end{aligned} \tag{17}$$

Furthermore, letting P_E^I be the well-known upper bound of the so-called event error probability of $\tilde{Y}^I$ for the ordinary Viterbi decoding, i.e.

$$P_E^I \leqq \sum_{L=1}^{\infty}\sum_{\hat{X}_L} Q\left(\sqrt{\frac{2I^2 d}{\sum_{i=1}^{I}\frac{1}{\eta_I}}}\right), \tag{18}$$

The block error probability $P(E_I)$ is bounded as follows.

$$P(E_I) \leqq 1 - \left(1 - P_E^I\right)^{N-\kappa}, \tag{19}$$

where κ denotes the constraint length of the convolutional code.

3. Performance Evaluation of PR-ADC Scheme

3.1 Normalized Average Number of Transmission

Since the length of retransmitted packet varies every retransmission in the PR scheme, the average number of transmission M_{PR} is no longer related to the throughput. Hence we use the normalized average number of transmitted packets $\overline{M}_{PR}$ defined as (*Expected number of all transmitted bits*)$/N$, which is related to the throughput τ_{PR} by $\tau_{PR} = R/\overline{M}_{PR}$. In order to evaluate $\overline{M}_{PR}$, we introduce the following notation.

- q_k^m k-th transmitted packet caused by the event that $\overline{\eta}_{k-1}$ of $\tilde{Y}^{k-1}$ is in retransmission region R_m. We represent the first transmitted packet by q_1.
- $\tilde{Y}^{I(ij\cdots k)}$ Combined packet for the case that packets $q_1 q_2^i q_3^j \cdots q_I^k$ are transmitted.
- $F_{I(ij\cdots k)}^l$ Event that $\overline{\eta}_I$ of $\tilde{Y}^{I(ij\cdots k)}$ is in retransmission region R_l, i.e. $T_{l-1} > \overline{\eta}_I \geqq T_l$. F_1^l represents the same event for the first transmission.
- $S_{I(ij\cdots k)}$ Event that $\overline{\eta}_I$ of $\tilde{Y}^{I(ij\cdots k)}$ is in acceptance region R_0, i.e. $\overline{\eta}_I \geqq T_0$. S_1 represents the same event for the first transmission.
- $a_l = N_l/N$ Packet length ratio normalized by the whole packet length N. N_l stands for the packet length corresponding to retransmission region R_l. Let $a_0 = 0$.

Then, $\overline{M}_{PR}$ can be bounded from Eqs. (23) and (25) in Ref. [8] as follows.

$$\overline{M}_{PR} \geqq 1+\sum_{i=1}^{K}(a_i-a_{i-1})\int_{-\infty}^{\infty}p(\eta_1)P(\overline{F}_1^i)d\eta_1 + \sum_{j=1}^{K}(a_j-a_{j-1})\iint_{-\infty}^{\infty}p(\eta_1\eta_2) \cdot P(\overline{F}_1^K)P(\overline{F}_{2(K)}^j)d\eta_1 d\eta_2+\cdots. \quad (20)$$

$$\overline{M}_{PR} \leqq 1+\sum_{i=1}^{K}(a_i-a_{i-1})\int_{-\infty}^{\infty}p(\eta_1)P(\overline{F}_1^i)d\eta_1 + \sum_{i=1}^{K}\iint_{-\infty}^{\infty}p(\eta_1\eta_2)\min\{P(F_1^i), \cdot\sum_{j=1}^{K}(a_j-a_{j-1})P(\overline{F}_{2(i)}^j)\}d\eta_1 d\eta_2+\cdots. \quad (21)$$

where $P(\overline{F}_{I(ij\cdots k)}^i)=\sum_{l=i}^{K}P(F_{I(ij\cdots k)}^l)$.

3.2 Retransmission Probability

In the PR scheme, the number of retransmission depends on the bit position of $\tilde{Y}^{I(ij\cdots k)}$. However, since the retransmitted bit positions are shifted every retransmission, the retransmission number of each bit position differs at most only one each other. We now consider the combined packet $\tilde{Y}^{I(ij\cdots k)}=\tilde{Y}^J$, where the superscript "J" means that each symbol $\tilde{y}_j^J$ of $\tilde{Y}^J$ is combined from at most J and at least $J-1$ received symbols. Then, the equivalent Gaussian noise for $\tilde{y}_j^J$ has variance $\frac{1}{2\hat{J}^2}\sum_{i=1}^{\hat{J}}\frac{1}{\eta_i}$ where $\hat{J}$ is equal to $J-1$ or J, which depends on its position j in the subblock of the packet, and η_i is the SNR of the i-th transmission for $\tilde{y}_j^J$. Let $n_{(a,b)}^{\hat{J}}$ be the equivalent Gaussian noise at the a-th $(a=1,2,\cdots,K)$ position in the b-th $(b=1,2,\cdots,U)$ subblock. Then the estimated averaged SNR becomes

$$\overline{\eta}_J=\frac{N\mathcal{E}}{2\sum_{a=1}^{K}\sum_{b=1}^{U}\left(n_{(a,b)}^{\hat{J}}-2\sqrt{\mathcal{E}}\cdot\epsilon(a,b)\right)^2}, \quad (22)$$

where $\epsilon(a,b)$ is defined as

$$\epsilon(a,b)=\begin{cases}1, & \text{if decoding bit error occurs at position (a,b)}\\ 0, & \text{otherwise.}\end{cases} \quad (23)$$

By substituting Eq. (22) into $P(\overline{\eta}_{J_C}<T_{l-1})$ and approximating the χ^2 distribution with the Gaussian distribution, we can get

$$P(\overline{\eta}_{J_C}<T_{l-1}) \approx Q\left\{\frac{\frac{N}{T_{l-1}}-U\cdot\left(\sum_{a=1}^{K}\frac{1}{\hat{J}_a^2}\sum_{i=1}^{\hat{J}_a}\frac{1}{\eta_{ai}}\right)}{\sqrt{2U\sum_{a=1}^{K}\left(\frac{1}{\hat{J}_a^2}\sum_{i=1}^{\hat{J}_a}\frac{1}{\eta_{ai}}\right)^2}}\right\} \triangleq \Theta(T_{l-1}). \quad (24)$$

Hence, similarly from Eqs. (9) and (14), $P(\overline{F}_{I(ij\cdots k)}^l)$ can be bounded as follows.

$$P(\overline{F}_{I(ij\cdots k)}^l) \gtrsim P(\overline{\eta}_{I_C}<T_{l-1}) \approx \Theta(T_{l-1}), \quad (25)$$

$$P(\overline{F}_{I(ij\cdots k)}^l) = \sum_{\overline{X}^I}P(\overline{X}^I)P(\overline{\eta}_I<T_{l-1}|\overline{X}^I) \lesssim P(E_{I(ij\cdots k)})\{1-\Theta(T_{l-1})\}+\Theta(T_{l-1}), \quad (26)$$

where $E_{I(ij\cdots k)}$ is the event that errors occur when $\tilde{Y}^{I(ij\cdots k)}$ is decoded by the ordinary Viterbi decoder. Furthermore $P(F_{I(ij\cdots k)}^l)$ can be bounded above as follows.

$$P(F_{I(ij\cdots k)}^l) = \sum_{\overline{X}^I}P(\overline{X}^I)P(T_l\leqq\overline{\eta}_I<T_{l-1}|\overline{X}^I) = P(\overline{F}_{I(ij\cdots k)}^l)-P(\overline{F}_{I(ij\cdots k)}^{l+1}) \lesssim P(E_{I(ij\cdots k)})\{1-\Theta(T_{l-1})\} +\Theta(T_{l-1})-\Theta(T_l). \quad (27)$$

In the PR scheme, the pairwise error probability for an incorrect path depends on the position where the incorrect path diverges from the correct path. Hence such dependency has cycle $W=K\cdot m/(\gcd(K,m))^2$ when a convolutional code with rate n/m is used, where $\gcd(K,m)$ is the greatest common divisor of K and m. On the other hand, $m(\tilde{Y}^J,X)-m(\tilde{Y}^J,\hat{X})$ has mean $-2\mathcal{E}d$ and variance $4\mathcal{E}\sum_{a=1}^{K}\frac{d_a}{\hat{J}_a^2}\sum_{i=1}^{\hat{J}_a}\frac{\mathcal{E}}{2\eta_{ai}}$, respectively, where d_a denotes the Hamming distance counted at the a-th bit position in every subblock and $d=\sum_{a=1}^{K}d_a$. Hence, $P(E_{I(ij\cdots k)})$ can be bounded by

$$P(E_{I(ij\cdots k)}) \leqq 1-\left(1-\frac{1}{W}\sum_{w=1}^{W}\sum_{L=1}^{\infty}\sum_{\hat{X}_{Lw}} Q\left\{\sqrt{\frac{2d^2}{\sum_{a=1}^{K}\frac{d_a}{\hat{J}_a^2}\sum_{i=1}^{\hat{J}_a}\frac{1}{\eta_{ai}}}}\right\}\right)^{N-\kappa} \quad (28)$$

3.3 Bit Error Rate of PR-ADC Scheme

It is difficult to derive the similar tighter bound of error rate for moderately time-varying channels as for slowly time-varying channels shown in Ref. [8]. However, we can derive a little loose but general bound as follows.

$$\begin{aligned}
P_E =& \int_{-\infty}^{\infty} p(\eta_1)P(S_1E_1)d\eta_1 \\
&+\sum_{i=1}^{K}\iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(F_1^iS_{2(i)}E_{2(i)})d\eta_1 d\eta_2 \\
&+\iiint_{-\infty}^{\infty}\sum_{i=1}^{K}\sum_{j=1}^{K} p(\eta_1\eta_2\eta_3) \\
&\cdot P(F_1^iF_{2(i)}^jS_{3(ij)}E_{3(ij)})d\eta_1 d\eta_2 d\eta_3+\cdots \\
=& \int_{-\infty}^{\infty} p(\eta_1)P(S_1)P(E_1|S_1)d\eta_1 \\
&+\sum_{i=1}^{K}\iint_{-\infty}^{\infty} p(\eta_1\eta_2) \\
&\cdot P(F_1^iS_{2(i)})P(E_{2(i)}|F_1^iS_{2(i)})d\eta_1 d\eta_2 \\
&+\sum_{i=1}^{K}\sum_{j=1}^{K}\iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P(F_1^iF_{2(i)}^jS_{3(ij)}) \\
&\cdot P(E_{3(ij)}|F_1^iF_{2(i)}^jS_{3(ij)})d\eta_1 d\eta_2 d\eta_3+\cdots \\
<& \int_{-\infty}^{\infty} p(\eta_1)P(S_1)P(E_1)d\eta_1 \\
&+\sum_{i=1}^{K}\iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(F_1^iS_{2(i)})P(E_{\mathrm{T}})d\eta_1 d\eta_2 \\
&+\sum_{i=1}^{K}\sum_{j=1}^{K}\iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P(F_1^iF_{2(i)}^jS_{3(ij)}) \\
&\cdot P(E_{\mathrm{T}})d\eta_1 d\eta_2 d\eta_3+\cdots \\
=& \int_{-\infty}^{\infty} p(\eta_1)P(S_1)P(E_1)d\eta_1 \\
&+\iint_{-\infty}^{\infty} p(\eta_1\eta_2)P(F_1S_2)P(E_{\mathrm{T}})d\eta_1 d\eta_2 \\
&+\iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)P(F_1F_2S_3) \\
&\cdot P(E_{\mathrm{T}})d\eta_1 d\eta_2 d\eta_3+\cdots \\
=& \int_{-\infty}^{\infty} p(\eta_1)\{1-P(F_1)\}P(E_1)d\eta_1 \\
&+\iint_{-\infty}^{\infty} p(\eta_1\eta_2) \\
&\cdot\{P(F_1)-P(F_1F_2)\}P(E_{\mathrm{T}})d\eta_1 d\eta_2 \\
&+\iiint_{-\infty}^{\infty} p(\eta_1\eta_2\eta_3)\{P(F_1F_2)-P(F_1F_2F_3)\} \\
&\cdot P(E_{\mathrm{T}})d\eta_1 d\eta_2 d\eta_3+\cdots \\
=& \int_{-\infty}^{\infty} p(\eta_1)\{P(E_1) \\
&+P(F_1)(P(E_{\mathrm{T}})-P(E_1))\}d\eta_1, \qquad (29)
\end{aligned}$$

where $P(E_{\mathrm{T}})$ is the block error probability of the ordinary Viterbi decoding for $\eta = T$ and $P(E_1)$ is the block error probability for Y^1. The first inequality comes from the fact that the condition $S_{I(ij\cdots k)}$ means that $\overline{\eta}_I$ of $\tilde{Y}^{I(ij\cdots k)}$ satisfies $\overline{\eta}_I \geqq T$ and hence the inequality $P(E_T) \geqq P(E_{I(ij\cdots k)}|F_1^iF_{2(i)}^j\cdots F_{I(ij\cdots l)}^kS_{I(ij\cdots k)})$ and $P(E_1) \geqq P(E_1|S_1)$ hold. Similarly, the bit error rate P_B is bounded by

$$P_B < \int_{-\infty}^{\infty} p(\eta_1)\{P_B^1+P(F_1)(P(B_{\mathrm{T}})-P_B^1)\}d\eta_1 \qquad (30)$$

where $P(B_{\mathrm{T}})$ is the bit error rate of the ordinary Viterbi decoding for $\eta = T$ and P_B^1 is the bit error rate for Y^1. We note that the bounds given by Eqs. (29) and (30) also hold for the both WR-ADC and WR-WDC schemes.

3.4 Determination of Retransmission Threshold

In the PR scheme, each thresholds T_l^I, $l = 0, 1, \cdots, K-1$, for the I-th transmission must be determined properly to prevent the repetition of partial retransmissions for the sake of the load reduction of the Viterbi decoder because each retransmission requires one Viterbi decoding. In the case of the slowly time-varying channels, the threshold T_l^I can be determined optimally to minimize the retransmission number as shown in Ref. [8]. Assume that I whole packets $q_1q_2^Kq_3^K\cdots q_I^K$ and one partial packet q_{I+1}^l are transmitted and $\tilde{Y}^{I+1(K\cdots Kl)}$ is combined from the corresponding received packets. Not to repeat the partial retransmission, the threshold T_l^I for $\tilde{Y}^{I(K\cdots K)}$ should be determined in such a way that $\tilde{Y}^{I+1(K\cdots Kl)}$ is accepted with probability almost one, i.e. $v \triangleq P(S_{I+1(K\cdots Kl)}) \approx 1$. From the relation

$$P(S_{I+1(K\cdots Kl)}) = 1 - P(\overline{F}_{I+1(K\cdots Kl)}^1) = v, \qquad (31)$$

we can determine the SNR η_v that satisfies Eq. (31). Hence, the threshold for $\tilde{Y}^{I(K\cdots K)}$ is obtained by $T_l^I = I\eta_v$ because each symbol of $\tilde{Y}^{I(K\cdots K)}$ is combined from I symbols, each SNR of which is η_v. For $v = 0.9$, these thresholds are given in Table 2. We refer this threshold scheme as T1 scheme.

In the case of moderately time-varying channels, the channel SNR varies every packet transmission. Hence, it is necessary to average Eq. (31) by the probability distribution of the channel SNR. Since each symbol $\tilde{y}_j^{I+1}$

Table 2 Retransmission threshold.

I	T_1^I(dB)	T_2^I(dB)	T_3^I(dB)
1	2.169	1.505	0.715
2	2.367	1.955	1.497
3	2.463	2.165	1.842
4	-	2.286	2.037
5	-	2.365	2.163
6	-	2.421	2.250

of $\tilde{Y}^{I+1(K\cdots Kl)}$ is combined from I or $I+1$ symbols, the variance of $\tilde{y}_j^{I+1}$ is given by $\frac{\mathcal{E}}{2I^2}\sum_{i=1}^{I}\frac{1}{\eta_i} \triangleq \frac{\mathcal{E}}{2I}\frac{1}{\xi^I}$ or $\frac{\mathcal{E}}{2(I+1)^2}\sum_{i=1}^{I+1}\frac{1}{\eta_i} = \frac{\mathcal{E}}{2(I+1)^2}\left(\frac{I}{\xi^I}+\frac{1}{\eta_{I+1}}\right)$, respectively. Furthermore, the numbers of $\tilde{y}_j^{I+1}$ constructed from I and $I+1$ symbols are $N - U \cdot l$ and $U \cdot l$, respectively. From Eq. (25), the acceptance probability $P(S_{I+1(K\cdots Kl)})$ for the moderately time-varying channels is given by

$$
\begin{aligned}
&P(S_{I+1(K\cdots Kl)}) \\
&= 1 - \int_{-\infty}^{\infty} p(\eta_{I+1}) P(\overline{F}^{1}_{I+1(K\cdots Kl)}) \\
&\approx 1 - \int_{-\infty}^{\infty} p(\eta_{I+1}) \\
&\cdot Q\left[\frac{\frac{N}{T_0} - \left\{\frac{N-U\cdot l}{I^2}\frac{I}{\xi^I} + \frac{U\cdot l}{(I+1)^2}\left(\frac{I}{\xi^I}+\frac{1}{\eta_{I+1}}\right)\right\}}{\sqrt{2\left\{\frac{(N-U\cdot l)}{I^4}\left(\frac{I}{\xi^I}\right)^2 + \frac{U\cdot l}{(I+1)^4}\left(\frac{I}{\xi^I}+\frac{1}{\eta_{I+1}}\right)^2\right\}}}\right] \\
&\cdot d\eta_{I+1}. \qquad (32)
\end{aligned}
$$

From Eq. (32), we can determine ξ_v^I that satisfies $P(S_{I+1(K\cdots Kl)}) = v$. Since the SNR of the equivalent channel such that $\tilde{Y}^{I(K\cdots K)}$ is received when X is transmitted is $I\xi_v^I$, the threshold for $\tilde{Y}^{I(K\cdots K)}$ should be $T_l^I = I\xi_v^I$. We refer this threshold scheme as T2 scheme.

We now apply T1 and T2 threshold schemes to the moderately time-varying channels although T1 threshold scheme is designed for slowly time-varying channels. The average number of transmission for these two threshold schemes, which are obtained by computer simulations, are shown in Fig. 2. In these simulations, we use $v = 0.9$. These results show that T1 threshold scheme can almost attain the same performance as T2 threshold scheme. T1 threshold scheme can be used without any prior information about the distribution of the channel SNR, but T2 threshold scheme requires it. Therefore, the T1 threshold scheme is practically preferable even for moderately time-varying channels.

4. Numerical Results

In this section, we compare the performance of WR-ADC and PR-ADC-T1 schemes by the theoretical bounds and simulation results. We used a convolutional code with generator polynomial $G = (1 + D + D^2, 1 + D^2)$, i.e. the constraint length is three and the rate is 1/2. The packet length N is assumed to be 1000 bits and the threshold T is set to 2.5 dB, which corresponds to $P_B \approx 3.0 \times 10^{-5}$. In the PR-ADC scheme, the length of subblock is four and the thresholds T_l^I are given by Table 2. As a moderately time-varying channel model, we assume that the SNR of the channel has a uniform distribution with width 2 dB, 4 dB or 6 dB or the Gaussian distribution with variance 0.5 dB, 1 dB or 2 dB. The

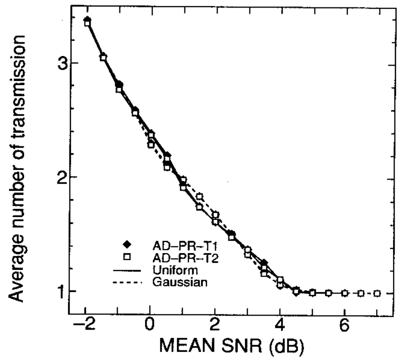

Fig. 2 Simulation result of the average number of transmission (uniform distribution with width 4 dB and Gaussian distribution with variance 1 dB).

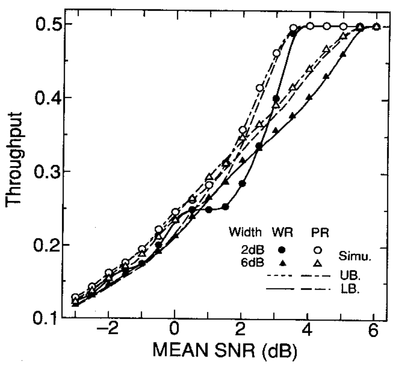

Fig. 3 Throughput of WR-ADC and PR-ADC scheme (uniform distribution).

integrations including Q-function, e.g. Eqs. (6), (7), etc., are calculated numerically.

In Figs. 3 and 4, the throughput are depicted for WR-ADC and PR-ADC-T1 schemes. From these results, we note that the throughput can be improved by using the PR-ADC scheme compared with the WR-ADC scheme for both uniform distributions and Gaussian distributions. Especially, the throughput is considerably improved by the PR-ADC scheme when the mean SNR is near the threshold T. This is caused from the fact that if SNR is near T, then the short partial retransmission occurs with high probability. We also note that the theoretical upper bound gives a good approximation.

The throughput of the PR-ADC scheme can be improved by increasing the length K of subblock because the retransmission length can finely be adjusted based on the estimated SNR. However, the larger the length of subblock K becomes, the more bits must be transmitted in feedback channel to inform the necessary retransmission length. This means that the error control for the feedback channel becomes severe. Furthermore, in the moderately time-varying channels, the very fine adjust-

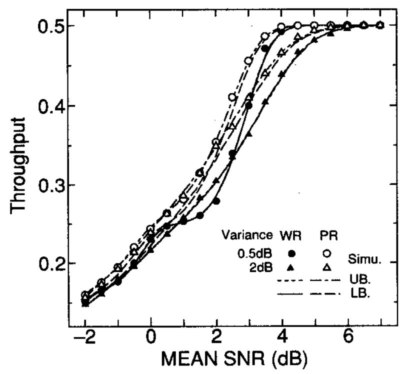

Fig. 4 Throughput of WR-ADC and PR-ADC scheme (Gaussian distribution).

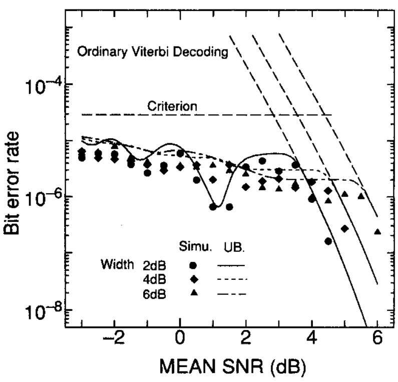

Fig. 5 Bit error rate of WR-ADC scheme (uniform distribution).

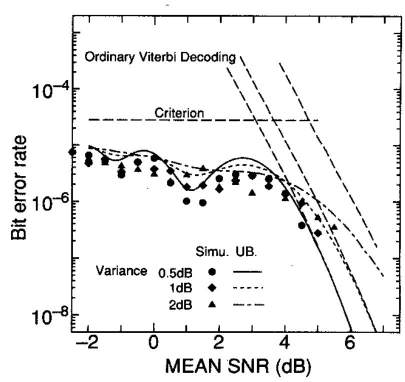

Fig. 6 Bit error rate of WR-ADC scheme (Gaussian distribution).

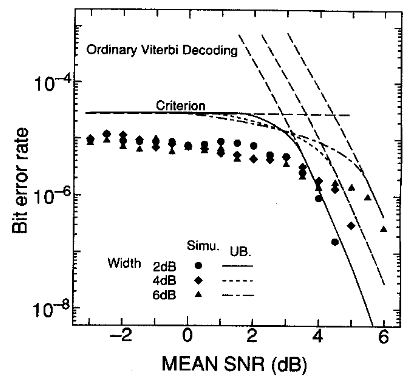

Fig. 7 Bit error rate of PR-ADC scheme (uniform distribution).

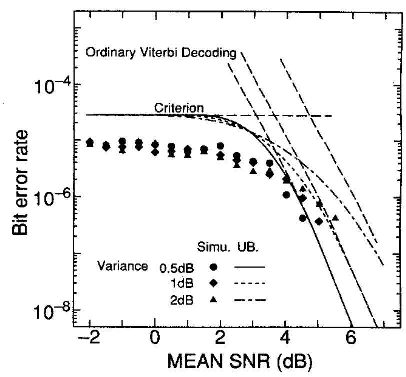

Fig. 8 Bit error rate of PR-ADC scheme (Gaussian distribution).

ment of retransmission length may be meaningless because the SNR varies every transmission. In the same way as the slowly time-varying channels [8], $K = 4$ or 8 is almost sufficient.

The performance of the bit error rate is depicted in Figs. 5 and 6 for the WR-ADC scheme while it is shown for the PR-ADC-T1 scheme in Figs. 7 and 8. From these results, we note that the both WR-ADC and PR-ADC schemes can attain the given bit error rate tolerance for moderately time-varying channels with any mean and any variance of SNR.

5. Conclusion

We treated the hybrid-ARQ scheme with retransmission criterion based on an estimated decoding error rate and we evaluated the performance by the theoretical bounds and computer simulations for moderately time-varying channels. As the packet combining schemes, we considered the ADC and WDC schemes. From the comparison of these two schemes, we showed that the ADC scheme can almost attain the same throughput as the optimal WDC scheme. Therefore, the ADC scheme is better because in the WDC scheme, the extra load of the Viterbi decoder is required to obtain the estimated variance of the channel noise.

For the PR-ADC scheme, we compared two threshold schemes. The one is designed for time-invariant channels while the other is designed for moderately time-varying channels by using the distribution of the channel SNR. By the simulation results, we showed that the information about the distribution of the channel

SNR little improve the average number of transmission, which is proportional to the load of the Viterbi decoder, and hence the threshold scheme for slowly time-varying channels can be used for moderately time-varying channels, too. We also showed by theoretical bounds and computer simulations that the PR-ADC scheme can improve the throughput compared with the WR-ADC scheme, and all schemes we consider can attain the given bit error rate tolerance for moderately time-varying channels with any mean and any variance of SNR.

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Appendix

The ADC scheme is not optimal especially for $p(\eta)$ with large variance, and the SNR of the nonoptimally combined $\tilde{Y}_{AD}$ becomes worse than the optimally combined $\tilde{Y}_{WD}$. But this does not mean that the bit error rate of the ADC scheme becomes worse than the WDC scheme because if the estimated SNR $\overline{\eta}_I$ is less than the threshold T, then retransmission is requested, i.e. no error occurs for $\overline{\eta}_I < T$. Actually, the bit error rate is bounded by Eq. (30) for both ADC and WDC schemes. This bound and simulation results, which are omitted, show that both ADC and WDC schemes can attain the given error rate tolerance for any $p(\eta)$. Hence the performance we must mainly compare is the throughput.

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